Let $\mathcal{H}$ be an infinite dimensional separable Hilbert space, $\mathcal{U}(\mathcal{H})$ the unitary group acting on $\mathcal{H}$, $G$ a complete separable metric group and $\phi : G \to \mathcal{U}(\mathcal{H})$ an algebraic isomorphism. Then $\phi$ is a topological isomorphism. The theorem is false for $\mathcal{U}(\mathcal{H})$ if $\mathcal{H}$ is finite dimensional. (Received September 18, 2007)