The purpose of this paper is to introduce numerical optimization methods in a Banach space setting that uses the idea of a gradient defined for non-Hilbert Banach spaces. The gradient is that of Michael Golomb and Richard A. Tapia, The Metric Gradient in Normed Linear Spaces, Numer. Math. 20, 115-124 (1972). This approach is motivated by introducing the notion of a Sobolev local minimum in the calculus of variations and relating it to a weak local minimum. The gradient can be computed for a Frechet differentiable functional defined on a non-Hilbert Sobolev space; this can be done by extending the Reisz Representation Theorem for Lp to a representation theorem for a continuous linear functional on a Sobolev space. This approach has applications to the calculus of variations and differential equations. Some of the gradient methods are described below. There are two methods given for a differential equation of steepest descent. One approach for a numerical solution of a differential equation in this setting is an extension of the Adams Bashforth method. Also conjugate gradient methods are presented with local convergence criteria. One example and application relates to Plateau’s Problem. (Received September 16, 2007)