Spaces of trees, spaces of graphs, and spaces of embeddings.

We will discuss how Michael Weiss’ orthogonal calculus can be used to study spaces of smooth embeddings. This is the subject of a long-term investigation that is still ongoing. We will survey the progress to date, and possibly will mention some questions that are currently under investigation. For example, we will explain how certain spaces of forests arise in the derivatives of stable homotopy of embedding spaces and how spaces of connected graphs arise in the unstable case. The Taylor towers split (rationally) in case of embeddings into a Euclidean space of high enough dimension. This splitting result relies on a theorem of Kontsevich to the effect that the little disks operad is formal. The splitting result can be refined further (to a double splitting result) in the case of embeddings of $\mathbb{R}^m$ into $\mathbb{R}^n$ that are standard outside the unit ball. This refined splitting result can be used to give rather explicit ”models” for the rational homology and homotopy of these spaces of embeddings, at least in large enough codimensions.

The splitting results are joint work with Pascal Lambrechts, Victor Turchin, and Ismar Volic. (Received September 19, 2007)