Quantum field theory (QFT) has entered many branches of modern mathematics. In some instances, new mathematical structures, for example Calabi-Yau categories, were discovered by studying quantizations proposed by physicists. In other instances, easy types of QFT’s, for example those that are topological, lead to new invariants of low-dimensional manifolds. In this talk I will report on a joint project with Stephan Stolz that attempts to define the ”space” of all QFT’s and interpret it in terms of a classifying space for generalized cohomology theories.

I will first discuss how to extend Segal’s definition of a QFT over a manifold X in various directions, most significantly to super symmetric QFT’s. Then the resulting theories for (d|1)-dimensional space-time will be explained in the cases d=0,1,2. It turns out that d=0 leads to closed differential forms on X, d=1 to certain vector bundles with connection over X, and that a 2-dimensional susy QFT gives an integral modular form. Taking concordance classes of such QFT’s over X leads to three basic (generalized) cohomology theories for d=0,1,2: de Rham cohomology, K-theory and, conjecturally, the theory of topological modular forms, the universal ”elliptic” cohomology theory introduced by Hopkins and Miller. (Received September 15, 2007)