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**Jeremy Avigad\*** (avigad@cmu.edu), Department of Philosophy, Baker Hall 135, Carnegie Mellon University, Pittsburgh, PA 15213. *Computability in Ergodic Theory*.

Let  $T$  be a measure-preserving transformation of a space  $(X, \mathcal{B}, \mu)$ , let  $f$  be a measurable function from  $X$  to  $\mathbb{R}$ , and for every  $x \in X$  and  $n \in \mathbb{N}$  let  $(A_n f)x = (fx + f(Tx) + \dots + f(T^{n-1}x))/n$ . The pointwise ergodic theorem says that this sequence of averages converges for almost every  $x$ , and the mean ergodic theorem says that the sequence  $(A_n f)$  converges in the  $L^2$  norm.

In general, one cannot compute a rate of convergence from the initial data. Describing joint work with Philipp Gerhardy and Henry Townser, I will explain how proof-theoretic methods provide classically equivalent formulations of the ergodic theorems which are computably valid, and yield additional information. Specifically, the mean ergodic theorem is equivalent to the assertion that for every function  $K(n)$  and every  $\varepsilon > 0$ , there is an  $n$  with the property that the ergodic averages  $A_m f$  are stable to within  $\varepsilon$  on the interval  $[n, K(n)]$ . Even in situations where the sequence  $(A_n f)$  does not have a computable limit, one can give explicit bounds on such  $n$  in terms of  $K$  and  $\|f\|/\varepsilon$ . These bounds can be used to obtain a similarly explicit version of the pointwise ergodic theorem as well. (Received September 14, 2008)