On a Conjecture Regarding the Coefficients of Cyclotomic Polynomials.

Let $a_n(k)$ be the coefficient of $x^k$ in the $n$th cyclotomic polynomial

$$
\Phi_n(x) = \prod_{j=1, \gcd(j,n)=1}^n (x - e^{2\pi i j/n})
$$

Let $M(a_n(k)) = \lim_{x \to \infty} \frac{1}{x} \sum_{n \leq x} a_n(k)$ be the average of $a_n(k)$, as introduced by Möller, and let

$$
f_k = \frac{\pi^2}{6} M(a_n(k)) k \prod_{\substack{q \leq k \\ q \text{ prime}}} (q + 1).
$$

It was conjectured by Y. Gallot, P. Moree and H. Hommersom that the $f_k$ are integers for all $k$. In this paper, we prove this conjecture. Moreover, we show that for any fixed natural number $n$, $f_k$ contains $n$ as a factor for sufficiently large $k$. (Received September 15, 2008)