The coefficients of the Fourier series of modular forms often have interesting number-theoretic properties. Modular forms for congruence groups have “bounded denominators” in the sense that if all of the Fourier coefficients are algebraic numbers (roots of integer polynomials), then they can be multiplied by some constant so that they are all algebraic integers (roots of monic integer polynomials). It is a conjecture that for noncongruence groups the opposite is true: Every purely noncongruence modular form has “unbounded denominators”. In this talk we consider the conjecture for a special class of noncongruence groups, character groups, which are normal subgroups of congruence groups with finite abelian quotient. We show how to construct families of character groups for which the Unbounded Denominator Conjecture is true. (Received September 16, 2008)