A conjecture of Rudnick and Ailon asserts that for multiplicatively independent integers $a > 1$ and $b > 1$, there are infinitely many exponents $n \geq 1$ such that $\gcd(a^n - 1, b^n - 1) = \gcd(a - 1, b - 1)$. We present experimental evidence and a heuristic argument for the statement that the number of primes $p < X$ such that $\gcd(a^p - 1, b^p - 1) = \gcd(a - 1, b - 1)$ is equal to $\pi(X)(1 + O(1/\log X))$. We will also discuss generalized versions of the Rudnick–Ailon conjecture for elliptic curves and other algebraic groups. (Received August 27, 2008)