1046-11-811
Kelley Harris* (`harris.kelley@gmail.com`), 213 Quincy Mail Center, 58 Plympton St, Cambridge, MA 02138. *On Integers n that Divide $\phi(n) + \sigma(n)$.*

The expressions $\phi(n) + \sigma(n) - 3n$ and $\phi(n) + \sigma(n) - 4n$ are unusual among linear combinations of arithmetic functions in that they each vanish on a nonempty set of composite numbers. In 1966, Nicol proved that the set $\mathcal{A} = \{n \mid (\phi(n) + \sigma(n))/n \in \mathbb{N}_{\geq 3}\}$ contains $2^a \cdot 3 \cdot (2^{a-2} \cdot 7 - 1)$ if and only if $2^{a-2} \cdot 7 - 1$ is prime and conjectured that $\mathcal{A}$ contains no odd integers. In this paper, we let $\mathcal{A}_K$ denote the set of $n \in \mathcal{A}$ with exactly K prime factors and present an algorithm that decides whether Nicol's conjecture holds for a given $\mathcal{A}_K$. We verify Nicol's conjecture for numbers with fewer than seven prime factors, and completely classify the elements of $\mathcal{A}$ that have fewer than five prime factors. In addition, we prove that every $\mathcal{A}_K$ is contained in a finite union of sequences that each converge with respect to some p-adic norm, and that the elements of $\mathcal{A}_4$ and $\mathcal{A}_5$ are contained in a 2-adically convergent sequence. (Received September 11, 2008)