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In earlier papers, we have considered the property for a commutative unital ring of having only finitely many (unital) subrings. This study concludes here with the characterization of rings having only finite chains of subrings. In fact, we prove that these two properties are equivalent, along with the following result. A commutative unital ring R with prime ring F has only finite chains of subrings if and only if R has only finitely many subrings, if and only if $R = F[t_1, \dots, t_n]$, where $F[t_i]$ has only finite chains of subrings for each i , if and only if $R = F[t_1, \dots, t_n]$, where $F[t_i]$ has only finitely many subrings for each i . A sufficient condition for the existence of an infinite chain of (unital) subalgebras is as follows. Let $R \subset T$ be a ring extension such that T is a finitely generated R -module, R is not a total quotient ring and $(R : T) = 0$. Then there exists a denumerable chain of R -subalgebras of T . (Received September 12, 2008)