Abelian formulas for cyclic curves.

Generalized elliptic curves called $C_{a,b}$ curves, namely $f(x,y) = y^a + x^b + f_{a-1,b-1}(x,y)$ ($a$ and $b$ coprime positive integers and in $f_{a-1,b-1}(x,y)$, a monomial $x^ry^s$ satisfies $ar + bs < ab$), have emerged in areas as different as number theory and PDEs. Generalizing the genus-1 equation (Kiepert) for the nonzero points of period $n$, the determinant $\sigma(nz)/\sigma(z)^{n^2}$ of a matrix with entries derivatives of the $\wp$ function, we give a determinantal equation for a polynomial in $(x,y)$ that vanishes at a point $P$ of a $C_{a,b}$ curve if and only if the Abel image of $nP$ belongs to the theta divisor $\Theta$. We use addition theorems generalizing Klein-Baker’s work on the higher-genus Weierstrass $\sigma$ function (J.C. Eilbeck, V.Z. Enolski, S. Matsutani, Y. Ônishi and E. Previato, *Int. Math. Res. Not.* 2008). For such curves with a specific group action we give more refined statements on the stratification of $\Theta$. For a genus-$3$ curve that admits an automorphism of order $3$ with quotient $\mathbb{P}^1$ we find formulas that generalize Jacobi’s $sn^2(z) + cn^2(z) = 1$ (for a hyperelliptic curve, cf. S. Matsutani, *Surv. Math. Appl.* 2008). (Received September 14, 2008)