Let $X$ denote the curve $y^m = x^p - x$ over a field of characteristic $p$. It is known that the automorphism group $G$ of $X$ is an extension of $\mathbb{Z}/m\mathbb{Z}$ by $PGL(2, p)$. Let $D$ be a $G$-invariant divisor on $X$. We compute explicitly the $G$-module structure of the Riemann-Roch space $L(D)$ (equivalently, on the linear system $|D|$). Examples using SAGE are given to illustrate both the computational nature of the results, and the applications to the theory of error-correcting codes. (Received September 16, 2008)