The control points and weights of a Bézier patch in geometric modeling provide an intuitive means to control the shape of the patch. Through de Casteljau’s algorithm, the control points enable the computation of the patch. Finer aspects of the patch, particularly continuity and smoothness along the boundary between two patches are also determined by the control points. Global properties, such as the location of a patch in space, are also governed by the control points.

We use methods from toric geometry to explain how some further global properties of a patch are governed by the control points. We define the control surface, which is an analog of the control polygon of curves. This is a piecewise linear triangulated surface whose vertices are the control points. We show that regular control surfaces are the limits of patches as the weights undergo a toric deformation corresponding to the underlying regular triangulation. This gives a precise meaning to the notion that the shape of the control net governs the shape of the patch.

A second global property that we investigate is self-intersection. We give a simple condition on a set of control points which implies that the resulting patch is injective, for any choice of weights. (Received August 26, 2008)