On the maximal order of an automorphism of a trigonal Riemann surface.

A closed Riemann surface $X$ which is a 3-sheeted covering of the Riemann sphere $f : X \to \hat{\mathbb{C}}$ is called trigonal and the covering $f$ is called a trigonal morphism. If $f$ is a cyclic covering, then $X$ is called cyclic trigonal. Otherwise $X$ is called a generic trigonal surface. Let $s$ be a singular value of $f$. If $f$ is cyclic, then $s$ is an order 3 singular value. If $f$ is non-cyclic, then $s$ is either a singular value of order three or a simple singular value. If all the singular values of $f$ are simple we say that $f$ is a simple covering. Simple coverings play an important role, for instance in the study of the moduli space. It is well known that the maximal order of an automorphism of a Riemann surface of genus $g$ is $4g + 2$. We study the maximal order of an automorphism of a trigonal Riemann surface. We find that the order of an automorphism of a cyclic trigonal Riemann surface of genus $g$, $g \geq 5$, is at most $3g + 3$ while the order of an automorphism of a generic trigonal surface of genus $g$, $g \geq 5$, is at most $2g + 1$. Finally we obtain that the order of an automorphism of a trigonal surface with simple morphism is at most $g + 1$. We show that the bounds above are sharp for infinite families of curves. (Received September 01, 2008)