Assymptotic Regularity: Are we almost at infinity yet? Preliminary report.

Suppose $I \subset S = k[x_0, \cdots, x_n]$ is a homogeneous ideal. A surprising theorem of Cutkosky-Herzog-Trung, Kodiyalam, and Trung-Wang asserts that for $t \gg 0$ the Castelnuovo-Mumford regularity of $I^t$ is a linear function of $t$, say $dt + e$. The invariant $d$ is relatively easy to identify, and in recent work Harris and I showed that, in a leading special case, the invariant $e$ is connected with the regularities of fibers of a related morphism of varieties. That left—in every case—the question, “How large does $t$ have to be?” I’ll explain the background, and discuss a recent result from joint work with Bernd Ulrich that gives a reasonably sharp bound in the special case I treated with Harris. The work leaves open some fundamental questions about Rees algebras, and I’ll discuss these as well. (Received September 06, 2008)