Richard Brualdi, Louis Deaett, Luz DeAlba, Jason Grout* (grout@iastate.edu), In-Jae Kim, Steve Kirkland, Raphael Loewy, Judith McDonald, Pauline van den Driessche and Amy Yielding. Minimum rank of graph powers. Preliminary report.

For a simple undirected graph $G$, let $G^r$, the $r$th graph power of $G$, be the graph with the same vertices as $G$ and edges

$$\{(u,v) \mid \text{there exists a walk of length at most } r \text{ from } u \text{ to } v \text{ in } G\}.$$

The minimum rank of a simple undirected graph with $n$ vertices is the minimum rank over all real symmetric $n \times n$ matrices with nonzero entries corresponding to the edges of the graph (the diagonal entries are not restricted). We compute the minimum rank and realizing matrices for the graph powers of paths. We also define a variant of $G^r$ with edges

$$\{(u,v) \mid \text{there exists a walk of length exactly } r \text{ from } u \text{ to } v\}$$

and again compute minimum rank and realizing matrices of powers of paths. We also obtain some results for minimum ranks of our variant powers of cycles and trees.

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