1046-15-440 **Abdel Nasser El-Kassar*** (abdulnasser.kassar@lau.edu.lb), Mme Curie Street, P.O. Box 13-5053, Beirut, 1102-2801, Lebanon, and **Samer S. Habre** and **Yehia Awad**. *GCD and LCM Matrices on Factor Closed Sets Defined over Principal Ideal Domains.*

Let $S = \{x_1, x_2, ..., x_n\}$ be a set of n distinct positive integers and let $s_{ij} = (x_i, x_j)$ be the greatest common divisor of x_i and x_j. The matrix $[S] = (s_{ij})$ is called the greatest common divisor (GCD) matrix on S. The matrix $[[S]]$ having its i, j-entry as the least common multiple of x_i and x_j is called the least common multiple (LCM) matrix on S. A set S is said to be factor closed (FC) if it contains every divisor of x for any $x \in S$. Smith (1875) showed that the determinant of $[S]$ on a FC set S is the product $\prod_{i=1}^{n} \phi(x_i)$, where ϕ is Euler's totient phi-function. Moreover, he obtained a formula for the determinant of $[[S]]$ on a FC set. In 1989, Beslin and Ligh obtained a structure theorem for GCD matrices and generalized the Smith's determinant to factor closed sets.

Since then many results concerning GCD and LCM matrices have been published.

In this paper, we generalize the notions of GCD and LCM matrices to any principal ideal domain (PID). Many of the important results, such as structure theorems, converse of Smith's result, determinants of reciprocal GCD matrix, inverse GCD matrix, and LCM matrix, are extended to such domains. (Received September 03, 2008)