Informally speaking, the essential dimension of an algebraic object (e.g., a finite-dimensional algebra or a quadratic form) is the minimal number of independent parameters one needs to define it. This notion has rich connections to various classical problems in algebra, including the algebraic form of Hilbert’s 13th problem, Albert’s cyclicity problem, Serre’s Conjecture II, and the Serre-Grothendieck theory of “special groups”.

In the context of central simple algebras this invariant first came up in a 1967 paper of C. Procesi, who showed that the essential dimension of a central simple algebra of degree $n$ is bounded from above by $n^2$. Computing the exact value for a generic division algebra of degree $n$ remains an open problem. In this talk I will discuss known results in this area, including recent joint work with A. Meyer on the essential dimension of a pair $(A, L)$, where $A/K$ is a division algebra of degree $n$ and $L/K$ is a maximal subfield of $A$. (Received September 15, 2008)