Finite $W$-algebras are certain associative algebras associated to nilpotent orbits in semisimple Lie algebras. They are deformations of the universal enveloping algebra of the centralizer of the nilpotent orbit. In this talk I will explain how the finite $W$-algebras associated to a class of nilpotent orbits in classical Lie algebras are quotients of twisted Yangians. Using this and Molev’s classification of finite dimensional irreducible representations of twisted Yangians, we deduce the classification of finite dimensional irreducible representations of these finite $W$- algebras, and relate it to a general conjecture of Brundan, Goodwin and Kleshchev which is phrased in terms of primitive ideals. (Received September 16, 2008)