1046-30-981 **Michael Eisermann*** (Michael.Eisermann@ujf-grenoble.fr), Institut Fourier, Universite Grenoble 1, 100 rue des Maths, BP 74, 38402 St Martin d'Heres, France. *An effective proof of the Fundamental Theorem of Algebra via Sturm chains.*

Sturm's famous theorem provides an elegant algorithm to count and locate the real roots of any given real polynomial. It is less widely known that Cauchy extended this to an algebraic method to count and locate the complex roots of any given complex polynomial. We give an algebraic proof of this beautiful result, starting from the mere axioms of the fields $\mathbb{R}$ and $\mathbb{C}$, without any further appeal to analysis. From this we derive a real algebraic proof of the Fundamental Theorem of Algebra, stating that every complex polynomial of degree n has precisely n complex roots. The proof is constructive and provides an explicit root finding algorithm. The proof is elementary inasmuch as it uses only polynomial arithmetic and the intermediate value theorem for real polynomials in one variable. As a consequence, all arguments hold over an arbitrary real closed field. (Received September 13, 2008)