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**Andrea Bruder\*** ([Andrea\\_Bruder@baylor.edu](mailto:Andrea_Bruder@baylor.edu)), Baylor University, Department of Mathematics, One Bear Place #97328, Waco, TX 76798. *An application of the left-definite spectral theory to the Jacobi differential equation for non-classical parameters.*

In 1929, S. Bochner classified all second order equations of hypergeometric type that have orthogonal polynomial eigenfunctions. Up to a complex change of variable, the only such equations are the Hermite, Laguerre, Jacobi, and the Bessel polynomial equations. Since then, it has been well known that, for  $-\alpha, -\beta, -\alpha - \beta - 1 \notin \mathbb{N}$ , the Jacobi polynomials  $\left\{ P_n^{(\alpha, \beta)}(x) \right\}_{n=0}^{\infty}$  are orthogonal on  $\mathbb{R}$  with respect to a bilinear form of the type

$$(f, g)_{\mu} = \int_{\mathbb{R}} f \bar{g} d\mu,$$

for some measure  $\mu$ . However, for negative integer parameters  $\alpha$  and  $\beta$ , an application of Favard's theorem shows that the Jacobi polynomials cannot be orthogonal on the real line with respect to a bilinear form of this type for any measure. But it is known that they are orthogonal with respect to a Sobolev inner product.

After discussing this Sobolev orthogonality, I will give an introduction to the left-definite spectral theory and show how it can be applied to construct a self-adjoint operator that is generated from the Jacobi differential expression (for non-classical parameters) having the entire sequence of Jacobi polynomials as a complete set of eigenfunctions. (Received September 15, 2008)