Andrea Bruder* (Andrea_Bruder@baylor.edu), Baylor University, Department of Mathematics, One Bear Place #97328, Waco, TX 76798. An application of the left-definite spectral theory to the Jacobi differential equation for non-classical parameters.

In 1929, S. Bochner classified all second order equations of hypergeometric type that have orthogonal polynomial eigenfunctions. Up to a complex change of variable, the only such equations are the Hermite, Laguerre, Jacobi, and the Bessel polynomial equations. Since then, it has been well known that, for \(-\alpha, -\beta, -\alpha - \beta - 1 \notin \mathbb{N}\), the Jacobi polynomials \(\left\{P_n^{(\alpha,\beta)}(x)\right\}_{n=0}^{\infty}\) are orthogonal on \(\mathbb{R}\) with respect to a bilinear form of the type

\[(f,g)_\mu = \int_R f(x)g(x)d\mu(x),\]

for some measure \(\mu\). However, for negative integer parameters \(\alpha\) and \(\beta\), an application of Favard’s theorem shows that the Jacobi polynomials cannot be orthogonal on the real line with respect to a bilinear form of this type for any measure. But it is known that they are orthogonal with respect to a Sobolev inner product.

After discussing this Sobolev orthogonality, I will give an introduction to the left-definite spectral theory and show how it can be applied to construct a self-adjoint operator that is generated from the Jacobi differential expression (for non-classical parameters) having the entire sequence of Jacobi polynomials as a complete set of eigenfunctions. (Received September 15, 2008)