Bilinear equations, integrable semi-discretization, and novel numerical computations of the Camassa-Holm equation.

The Camassa-Holm (CH) equation has attracted considerable interest since it was derived as a model equation for shallow-water waves. The CH equation is shown to be completely integrable, admitting non-smooth solutions such as peakon and cuspon solutions. However, it is extremely difficult to perform numerical computations of the CH equation due to the singularities of the solutions. So far, none of the numerical methods for the CH equations gives a satisfactory result.

From a reduction of KP-Toda hierarchy, we first give bilinear equations directly related to the CH equation. Then, we propose an integrable semi-discrete CH equation. Determinant formulas for multi-soliton solutions of the continuous and semi-discrete CH equations are presented, from which smooth soliton and cuspon soliton solutions for both continuous and semi-discrete CH equations can be generated. From the point view of numerical analysis, a novel numerical method, i.e., a self-adaptive method is implied from the semi-discrete CH equation for the numerical computations CH equation. The results of numerical simulations are very surprising, showing very accurate numerical solutions for the soliton-soliton and cuspon-cuspon interactions by using coarse grid points. (Received September 15, 2008)