We construct a sequence \( \{a_n\} \) such that for any measure preserving system \((X, \Sigma, m, T)\) the ergodic averages

\[
A_Nf(x) = \frac{1}{N} \sum_{n=1}^{N} f(T^{a_n}x)
\]

converge a.e. for all \( f \) in \( L\log \log(L) \), but fail to have a finite limit for some \( L^1 \) function. In fact, we show that one may construct sequences for which these averages converge a.e. for any Orlicz space strictly smaller than \( L^1 \), but fail to converge for some \( L^1 \) function. This ultimately extends the work of K. Reinhold. Reinhold, building on the work of A. Bellow, constructed a sequence for which the averages \( A_Nf(x) \) converge a.e. for \( L^p \), \( p > 1 \), but do not converge for some \( f \) in \( L^1 \). Our method, introduced by Bellow and later extended by Reinhold and M. Wierdl, is perturbation. (Received September 17, 2008)