David Constantine* (constand@umich.edu), Department of Mathematics, 2074 East Hall, 530 Church St, Ann Arbor, MI 48109. On Compact Clifford-Klein Forms of $SL_{n-2}(\mathbb{R}) \backslash SL_n(\mathbb{R})$. Preliminary report.

The problem of compact Clifford-Klein forms is to determine all pairs of Lie groups $(H, J)$, where $J$ is a closed subgroup of $H$, which have a compact quotient $J \backslash H/\Gamma$ by a discrete subgroup of $H$ that acts properly discontinuously on $J \backslash H$. When $J$ is noncompact many cases of this problem are open. The basic case of $SL_{n-k}(\mathbb{R}) \backslash SL_n(\mathbb{R})$ is not completely solved; the main results are due to Zimmer and collaborators for $k \geq 3$ and to Benoist for $k = 1$ and $n$ odd, both showing that compact forms do not exist. In this talk I will present the following result for $k = 2$. Any compact form is given by the following construction: there is a subgroup $L$ of $SL_n(\mathbb{R})$ containing a cocompact lattice $\Lambda$ such that $SL_{n-2}(\mathbb{R}) \backslash SL_n(\mathbb{R})/\Gamma$ is naturally identified with $(L \cap SL_{n-2}(\mathbb{R})) \backslash L/\Lambda$. This confirms a remark by Margulis that all known constructions of compact forms for reductive $J$ are based on the existence of such a subgroup and reduces the compact form question to the algebraic question of whether such a subgroup $L$ exists. This is a preliminary report on ongoing research. (Received September 10, 2008)