1046-39-975 **Faruk F Abi-Khuzam*** (farukakh@aub.edu.lb), American University of Beirut, PO Box 11-0236, Riad El-Solh 1107 2020, Beirut, Lebanon. *Asymptotic behavior of solutions of a class of non-linear difference equations.* Preliminary report.

A first order linear difference equation can be solved explicitly, and a simple method for obtaining the solution may be given. In most applications, however, the relevant equation is usually non-linear and a closed form of the solution is, in general, impossible to obtain. This makes it necessary to find the asymptotic behavior of the solution. In this note we obtain the asymptotic behavior of the solution for a class of non-linear difference equations. For completeness we also include a derivation of a closed form of the solution of the general first order linear equation.

**Theorem 1.** If $f(n) \neq -1$ and $a_n$ is a solution of the difference equation

$$a_{n+1} - a_n - f(n)a_n = g(n)$$

then

$$a_{n+1} = a_1 \prod_{k=1}^{n}(1 + f(k)) + \sum_{k=1}^{n} g(k) \prod_{j=k+1}^{n}(1 + f(j))$$

where $a_1$ is arbitrary.

**Theorem 2.** Let $f$ be a positive non-decreasing function defined on $(0, \infty)$. Let $a_n$ be a solution of

$$a_{n+1} - a_n = \frac{1}{f(a_n)}$$

satisfying $a_1 > 0$. Put

$$F(x) = 1 + \int_0^x f(t)dt, x \geq 0.$$

Then

$$a_n \sim F^{-1}(n) \quad \text{as } n \to \infty.$$

**Example 3.** If $a_{n+1} - a_n = a_n^{-\alpha}$, where $a_1, \alpha > 0$, then $a_n \sim (\alpha+1)^{\frac{1}{\alpha+1}} n^{\frac{1}{\alpha+1}}$ as $n \to \infty$.

**Example 4.** If $a_{n+1} - a_n = \exp(-a_n)$, then $a_n \sim \ln n$ as $n \to \infty$. (Received September 13, 2008)