In this talk we will review some of the global asymptotic results obtained during the last two decades in the theory of the classical Painlevé equations with the help of the Isomonodromy - Riemann-Hilbert method. The results include the explicit derivation of the asymptotic connection formulae, the explicit description of linear and nonlinear Stokes phenomenon and the explicit evaluation of the distribution of poles. We will also discuss some of the most recent results emerging due to the appearance of Painlevé equations in random matrix theory. The Riemann-Hilbert method will be outlined as well. (Received September 15, 2008)