We extend the theory of local regularization for solving linear, first kind Volterra convolution equations with finitely smoothing kernels to allow for the underlying data spaces $L^p[0,1], 1 < p < \infty$. To do so, modifications must be made to the conditions established by P.K. Lamm for convergence with data in $C[0,1]$. This includes specifying appropriate families of signed measures appearing in the second-kind Volterra equation associated with local regularization and giving further conditions to guarantee well-posedness of the equation for all values of the regularization parameter in a designated interval. We prove that these modifications are sufficient to ensure that solutions to the second-kind Volterra equation, based on exact data, converge to the problem’s true solution in $L^p[0,1]$. We also provide an \textit{a priori} parameter selection strategy so that solutions, based on inexact data, converge to the problem’s true solution in $L^p[0,1]$ as the noise level and regularization parameter shrink to zero, i.e. the resulting local regularization method is $L^p$-convergent. Furthermore, we establish a rate of $L^p$-convergence of approximations satisfying the source condition of uniform Hölder continuity. (Received September 15, 2008)