An index formula related to a conjecture of Loewner.

We prove a geometric index formula which produces a “defect” term for the Loewner conjecture about the index of vector fields with isolated zeros of the form $\partial^n \bar{z} f$ for functions $f : \mathbb{C} \to \mathbb{R}$. A recent result of F. Xavier allows the index of such vector fields to be computed in terms of the set of eigenvalues of the Hessian of $f$ in the case $n = 2$. Our result extends this formula to all $n \geq 2$. The Loewner conjecture has a deep connection to the Carathéodory conjecture which states that a smooth, convex embedding of the 2-sphere into $\mathbb{R}^3$ has at least two umbilics. (Received September 12, 2008)