General affine surface areas.

In a joint work with Matthias Reitzner (Ann. of Math., to appear), we obtained the following classification of valuations on the space, $\mathcal{K}_0^n$, of convex bodies that contain the origin in their interiors.

**Theorem.** A functional $\Phi : \mathcal{K}_0^n \to \mathbb{R}$ is an upper semicontinuous and $\text{SL}(n)$ invariant valuation that vanishes on polytopes if and only if there is a concave function $\phi : [0, \infty) \to [0, \infty)$ with $\lim_{t \to 0} \phi(t) = \lim_{t \to \infty} \phi(t)/t = 0$ such that

$$\Phi(K) = \int_{\partial K} \phi(\kappa_0(K, x)) \, d\mu_K(x)$$

for every $K \in \mathcal{K}_0^n$.

Here $d\mu_K(x) = x \cdot u(K, x) \, dx$ is the cone measure on $\partial K$, $u(K, x)$ is the exterior normal unit vector to $K$ at $x \in \partial K$, and

$$\kappa_0(K, x) = \frac{\kappa(K, x)}{(x \cdot u(K, x))^{n+1}},$$

where $\kappa(K, x)$ is the Gaussian curvature.

In this talk, two new families of general affine surface areas are defined. Basic properties and affine isoperimetric inequalities for these new affine surface areas as well as for the $L_\phi$ affine surface areas defined in (1) are discussed. (Received September 15, 2008)