Compressed Sensing reconstruction algorithms typically exhibit a zeroth-order phase transition phenomenon for large problem sizes; there is a domain of problem sizes for which successful recovery occurs with overwhelming probability, and there is a domain for which recovery failure occurs with overwhelming probability. For $\ell^1$-regularization, this phenomenon is a manifestation of the number of low dimensional faces of randomly projected polytopes. These results give precise if and only if conditions on the number of samples needed in Compressed Sensing applications. (Received September 17, 2008)