We define the “blob complex” $B_*(M, C)$ associated to an $n$-manifold $M$ and a (suitable) $n$-category $C$. This is a simultaneous generalisation of two interesting gadgets. When $n = 1$, $M = S^1$ and $C$ is an algebra, the homology of the blob complex is the Hochschild homology of the algebra. On the other hand, the zero-th homology of the blob complex is the usual skein module of “pictures from $C$ drawn on $M$”. In this sense the blob complex is a “derived” variant of a TQFT.

I’ll give the definition, and explain a few important properties. We hope to apply blob homology to tight contact structures (for $n = 3$) and Khovanov homology (for $n = 4$). In both theories exact triangles play an important role. These exact triangles don’t interact well with the gluing structure of the associated TQFTs, however. Our motivation for considering blob homology is to work around these difficulties. (Received September 12, 2008)