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Roger W. Barnard (roger.w.barnard@ttu.edu), Department of Mathematics, Texas Tech University, Lubbock, TX 79409, **Michael B. Gordy*** (Michael.Gordy@frb.gov), Division of Research & Statistics, Board of Governors, Federal Reserve System, Washington, DC 20551, and **Kendall C. Richards** (richards@southwestern.edu), Department of Mathematics, Southwestern University, Georgetown, TX 78626. *A Turán Type Inequality for the Kummer Function Arising in Finance. Part I: The Application.*

When the value of a firm's assets V follows a geometric Brownian motion, the value of a perpetual debt contract as a function $F(V; \kappa)$ solves Kummer's differential equation with a boundary condition depending on the firm's bankruptcy threshold κ . We model a lending relationship in which covenants to the loan contract permit the bank to choose the bankruptcy threshold. The bank's optimal choice of κ solves a first order condition involving a ratio of contiguous Kummer functions. This implies that the Turán-type inequality for the Kummer function $\Phi(\alpha, \beta, x) \equiv \sum_{n=0}^{\infty} \frac{(\alpha)_n x^n}{(\beta)_n n!}$ arises naturally in studying the comparative statics of the model:

$$\Phi(a, a + b, x)^2 > \Phi(a + 1, a + b, x) \Phi(a - 1, a + b, x)$$

for all nonzero $x \in \mathbb{R}$ and $a, b > 0$. (Received August 13, 2008)