In 1900 David Hilbert posed 23 problems to the mathematical community. The tenth problem asked for an algorithm to decide whether an arbitrary polynomial equation with integer coefficients has a solution over the integers. In 1970 Yuri Matiyasevich proved that no such algorithm exists, completing earlier work of Martin Davis, Hilary Putnam, and Julia Robinson.

In this talk I will discuss some aspects of Matiyasevich’s proof. I will also talk about related problems, including the analogue of Hilbert’s Tenth Problem for equations with coefficients and solutions over the rational numbers. (Received September 10, 2008)