Examining the box topology on the Cartesian product of connected spaces. Preliminary report.

The product space of a family of topological spaces is the Cartesian product of these spaces, with a specific product topology. The basis of the product topology for $X = \prod_{i \in I} X_i$, where $I$ is an indexing set, is made up of sets of the form $\prod_{i \in I} U_i$ with $U_i$ an open subset of $X_i$ and $U_i = X_i$ for all but a finite number of indices.

One result of interest in the product topology is that if each component space $X_i$ is connected, the product space on $X = \prod_{i \in I} X_i$ is connected. However, this result holds specifically in the product topology. Another topology on the Cartesian product of sets, the box topology, is identical to the product topology on the same set, save without the restriction that $U_i = X_i$ for all but a finite number of indices. It is our goal to examine the box topology to see whether a space with the box topology is connected under the circumstance that each component space is in fact connected. We will accomplish this by considering the set $\prod_{i \in \mathbb{N}} \mathbb{R}$, where each component has a usual topology on $(\mathbb{R}, \mathcal{U})$. As $(\mathbb{R}, \mathcal{U})$ is connected, this set is connected under the product topology. We will examine whether this set is connected under the box topology. (Received September 12, 2008)