

1056-03-69

Fernando J. Ferreira* (ferferr@cii.fc.ul.pt), Universidade de Lisboa, Dep de Matemática, Faculdade de Ciências, Campo Grande, Ed. C6, Piso 2, P-1749-016 Lisboa, Portugal.

Bar-recursive interpretations of classical analysis.

Following the work of K. Gödel published in 1958, C. Spector gave in 1962 a remarkable characterization of the provably recursive functionals of full second-order arithmetic (a.k.a. analysis). Spector's interpretation relies on a form of well-founded recursion known as bar-recursion. Spector's interpretation is rather indirect, via the negative translation. We re-prove Spector's result avoiding the passage through intuitionistic logic, relying instead on the very simple direct interpretation of Peano arithmetic given by J. Shoenfield in 1967.

The bounded functional interpretation was introduced in 2005 by P. Oliva and the present author. In its version for classical arithmetic, it is an interpretation which “injects” some non set-theoretical uniformities into Peano arithmetic. Nevertheless, due to its Soundness Theorem, the interpretation is able to extract correct uniform bounds from certain theorems of mathematics (and this opens the way to possible contributions to Proof Mining).

We recently showed that the bounded functional interpretation is compatible (via bar-recursive functionals) with full numerical comprehension.

The above reported work is in collaboration with P. Engrácia. (Received September 15, 2009)