We propose a major index statistic on 01-fillings of moon polyominoes which, when specialized to certain shapes, reduces to the classical major index for permutations and set partitions. We consider the set $F(M, s; A)$ of all 01-fillings of a moon polyomino $M$ with given column sum $s$ whose empty rows are $A$, and prove that this major index has the same distribution as the number of north-east chains, which are the natural extension of inversions (resp. crossings) for permutations (resp. set partitions). Hence our result generalizes the classical equidistribution results for the permutation statistics inv and maj. Two proofs will be presented. The first is an algebraic one using generating functions, and the second is a bijection on 01-fillings of moon polyominoes in the spirit of Foata’s second fundamental transformation on words and permutations. (Received September 21, 2009)