Let $P$ be a fixed subposet of a Boolean lattice. Let the maximal number of elements in a Boolean lattice $Q_n$ that induce a subposet containing no copy of $P$ be $ex(n, P)$. Denote the size of a middle layer of $Q_n$ by $N$.

The classical Sperner theorem states that $ex(n, P_2) = N$, where $P_2$ is a two element chain. There are several other examples of posets for which the extremal function has been calculated asymptotically. In all of these known cases $ex(n, P) = iN(1 + o(1))$, where $i$ is an integer. It has been conjectured that the extremal function is always an integer multiple of the middle layer size.

The only poset with at most 4 elements for which this conjecture is not confirmed is $Q_2$. We provide improved bounds on $ex(n, Q_2)$ and show the limitation of classical methods applied to this problem. (Received September 21, 2009)