The $L(2, 1)$ channel-assignment problem on trees. Preliminary report.

Let $G = (V, E)$ be a simple graph. We say that a non-negative integer labeling $\ell$ of its vertices $V$ is called an $L(2, 1)$-labeling if for every pair $\{u, v\}$ of adjacent vertices $|\ell(u) - \ell(v)| \geq 2$, and for every pair $\{u, v\}$ satisfying $\rho(u, v) = 2$, $|\ell(u) - \ell(v)| \geq 1$, where $\rho$ is the usual path metric on $V$. (Such labelings model the assignment of non-interfering “channels” to nearby radio transmitters.) The $L(2, 1)$-span of a graph $G$, $\lambda(G)$, is defined to be the minimum value, over all $L(2, 1)$-labelings of $G$, of $\max_{v \in V} \ell(v)$.

In 1992 J.R. Griggs and R.K. Yeh proved that for a tree $T$ with maximal vertex degree $\Delta$, $\lambda(T) \in \{\Delta + 1, \Delta + 2\}$, but conjectured that for an arbitrary tree determining which of these values obtains would prove to be NP-hard.

We describe a deterministic algorithm for computing $\lambda(T)$ in the case $\Delta = 3$ and indicate how this algorithm can be generalized to arbitrary maximal degree $\Delta$. The algorithm has exponential time complexity, and its construction shows why no more efficient deterministic algorithm can be found. (Received July 21, 2009)