Hung-ping Tsao* (hptsao@hotmail.com), 1151 Highland Drive, Novato, CA 94949. Triangular arrays induced from trigonometric functions.

Using \( \cos 2nA = 2 \cos nA \cos nA - 1 \) and \( \cos (2n-1)A = 2 \cos nA \cos (n-1)A - \cos A \), we can obtain the polynomial expressions in \( \cos A \) for \( \cos mA \). Let \( c(m,k) \) denote the triangular array formed by the coefficients in question. Then we can use \( c(2n-1,2k) = c(2n,2k-1) = 0 \), \( c(4n-2,0) = -1 \), \( c(4n,0) = 1 \), \( c(1,1) = 1 \), \( c(2,2) = 2 \), \( c(2n+1,k) = 2c(2n,k-1) - c(2n-1,k) \) and \( c(2n+2,2k) = 2c(2n+1,2k-1) - c(2n,2k) \) to generate the entire array. The triangular array induced from \( \sin (2n-1)A \) is the same as that induced from \( \cos (2n-1)A \) except the signs for even \( n \). Let \( s(2n,k) \) be the triangular array induced from \( \sin 2(n+1)A / \cos A \). Then \( s(2n,2n-2k) \) is a linear combination of \( c(2n,2n-2k) \) and \( c(2n-2,2n-2) \) with the coefficients only involving powers of 4 and the binomial coefficient \( C(2n-k-1,k-2) \). Furthermore, we have \( s(2n,2k) = s(2n-2,2k) - 2(n-k+1)s(2n,2k-2)/k \). By writing \( \tan (2n-1)A \) and \( \tan 2nA \) into rational functions of \( \tan A \) and considering only nonzero entries, we obtain arrays \( t, u, v \) and \( w \) from numerators and denominators so that \( t \) and \( u \) are the same for odd \( n \) and differ in signs, where \( u(n,k) = C(2n-1,2k-2) \) for even \( n-k \) and \( u(n,k) = -C(2n-1,2k-2) \) for odd \( n-k \). We also have \( v(n,k) = C(2n,2k-1) \) for even \( k \), \( v(n,k) = -C(2n,2k-1) \) for odd \( k \), \( w(n,k) = C(2n,2k) \) for even \( k \) and \( w(n,k) = -C(2n,2k) \) for odd \( k \). \( c \) and \( u \) are also related. (Received May 27, 2009)