Let $p$ be a prime number and $f(x_1, \ldots, x_n)$ be a polynomial with coefficients in $\mathbb{Z}$, the ring of integers. Let $c_m$ denote the number of solutions of $f \equiv 0 \pmod{p^m}$ with $c_0 = 1$. Then the Poincaré Series $P_f(y)$ is the generating function

$$P_f(y) = \sum_{i=0}^{\infty} c_i y^i.$$ 

Denef proved that $P_f(y)$ is always a rational function. We explicitly compute $P_f(y)$ when $f$ is an arbitrary diagonal polynomial, extending results of Qing Han. This is a special case of our main work that deals with diagonal polynomials over certain UFD’s. We also present some new results that give a criterion for an element to be an $n^{th}$ power in a complete discrete valuation ring.

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