This is a survey of several exciting recent results in which techniques originating in the area known as additive combinatorics have been applied to give results in other areas, such as group theory, number theory and theoretical computer science.

We begin with a discussion of the notion of an approximate group and also that of an approximate ring, describing key results of Freiman-Ruzsa, Bourgain-Katz-Tao, Helfgott and others in which the structure of such objects is elucidated.

We then move on to the applications. In particular we will look at the work of Bourgain and Gamburd on expansion properties of Cayley graphs on $\text{SL}_2(\mathbb{F}_p)$ and the existence of a spectral gap for certain sets of generators in $\text{SU}(2)$, and at the work of Bourgain, Gamburd and Sarnak on nonlinear sieving problems.

This being a survey, all of the notions mentioned here will be introduced from scratch. Technical details will be kept to an absolute minimum. (Received September 22, 2009)