Several papers have considered the problem of finding, for a fixed $n \in \mathbb{Z}^+$, all integral solutions to $xyz = x + y + z = 1$ contained in fields of degree $n$ over $\mathbb{Q}$. Complete results are known for $n \leq 4$, but only partial results are known for $n = 5$. In particular, it is not known (or even conjectured) that, for $n = 5$, all of the solutions have been found.

In this talk, I will summarize the earlier findings for $n \leq 5$ and then discuss more recent work that has led to the discovery of further solutions to the equation in the quintic field case. I will also discuss the remaining roadblocks to completely solving the problem in this case. (Received September 17, 2009)