

1056-11-936

**Sam Elder\*** (same@caltech.edu), MSC 385, Pasadena, CA 91126. *Flat Cyclotomic Polynomials: A New Approach.*

The *height* of a polynomial is its greatest coefficient in absolute value. Polynomials of unit height are *flat*. The *cyclotomic polynomial*  $\Phi_n(x)$  is the minimal polynomial of any primitive  $n$ th root of unity. The *order* of  $\Phi_n(x)$  is the number of distinct odd primes dividing  $n$ . All cyclotomic polynomials of orders 0, 1 and 2 are flat, and some of orders 3 and 4 are flat as well.

In this paper, we build a new theory for analyzing the coefficients of  $\Phi_n(x)$  by considering it as a gcd of simpler polynomials. We first obtain a generalization of a result known as periodicity: If  $n$  is a positive integer and  $s$  and  $t$  primes such that  $n - \varphi(n) < s < t$  and  $s \equiv \pm t \pmod{n}$ , then  $\Phi_{ns}(x)$  and  $\Phi_{nt}(x)$  have the same height.

We also use this theory to provide two new families of flat cyclotomic polynomials. One, of order 3, was conjectured by Broadhurst: Let  $p < q < r$  be primes and  $w$  a positive integer such that  $r \equiv \pm w \pmod{pq}$ ,  $p \equiv 1 \pmod{w}$  and  $q \equiv 1 \pmod{wp}$ . Then  $\Phi_{pqr}(x)$  is flat. The other is the first general family of order 4. We prove that  $\Phi_{pqrs}(x)$  is flat for primes  $p, q, r, s$  where  $q \equiv -1 \pmod{p}$ ,  $r \equiv \pm 1 \pmod{pq}$ , and  $s \equiv \pm 1 \pmod{pqr}$ . (Received September 18, 2009)