A great number of geometric and combinatorial properties of a given linear endomorphism $X$ of $\mathbb{R}^N$ is captured in the study of its associated zonotope $Z(X)$, and, by duality, its associated hyperplane arrangement $H(X)$. Of particular interest in various applications is the case $n \ll N$. We perform this study at an algebraic level, and associate $X$ with three algebraic structures, referred as external, central, and internal. Each algebraic structure is given in terms of a pair of homogeneous polynomial ideals in $n$ variables that are dual to each other: one encodes properties of the arrangement $H(X)$, while the other encodes by duality properties of the zonotope $Z(X)$. The algebraic structures are defined purely in terms of the combinatorial structure of $X$, but are subsequently proved to be equally obtainable by applying suitable algebraic or analytic operations to either of $Z(X)$ or $H(X)$. The theory is universal in the sense that it requires no assumptions on the map $X$, and provides new tools that can be used in enumerative combinatorics, graph theory, representation theory, polytope geometry, and analysis. (Received September 22, 2009)