An $A$-hypergeometric system is a system of PDEs determined by a toric ideal and certain homogeneity parameters. The dimension of its solution space, called its rank, is constant for generic parameters. I will discuss the combinatorial nature of this rank at non-generic parameters and its ties to the local cohomology of the toric algebra with support in the maximal ideal. (Received September 18, 2009)