The Kirchberg-Phillips theorem completely classifies purely infinite simple unital Cuntz-Kreiger graph $C^*$-algebras via $K$-theory, but does not describe the resulting isomorphisms explicitly. The classification of the purely algebraic Leavitt path algebras is as yet incomplete, but the explicit isomorphisms given in that classification extend to graph $C^*$-algebras as well. For example, isomorphisms between Leavitt algebras $L_n$ and their matrix rings also yielded the first explicitly described isomorphisms between the Cuntz algebras $\mathcal{O}_n$ and their matrix rings.

Recent work of the authors with Adel Louly and Enrique Pardo has achieved an algebraic analogue of the Kirchberg-Phillips theorem, classifying nearly all purely infinite simple unital Leavitt path algebras. We show that the resulting isomorphisms can be explicitly described, and the authors have implemented computer software to do so. As a result, nearly all isomorphisms implied by the Kirchberg-Phillips theorem for purely infinite simple unital graph $C^*$-algebras can be explicitly described as well. We also discuss the properties of these isomorphisms, and the obstacle to extending the classification result to all purely infinite simple unital Leavitt path algebras. (Received September 18, 2009)