We describe a basis theory on the edge set of a directed graph. This is used to study homomorphisms from directed graphs to groups and provides the foundation for constructing group-graded matrix algebras composed of blocks associated to components of directed graphs. Our results are related to an earlier construction of A. V. Kelarev for blocked matrix Lie superalgebras. (Received September 17, 2009)