Let $G$ be a finite group and $\mathbb{F}$ be a finite field. A projective indecomposable $\mathbb{F}G$-module (P.I.M.) is an indecomposable direct summand of the group algebra $\mathbb{F}G$. Computing the P.I.M.s of large finite groups has been always a challenging problem due to the large sizes of the representations of these groups. The theory of Morita and condensation are used for constructing the P.I.M.s of some finite groups, however there are limitations on the sizes of the representations that these techniques can be applied to. This paper describes a new method for constructing the P.I.M.s of large finite groups. The power of the algorithm is illustrated by the examples of the socle series of all P.I.M.s of the sporadic simple Mathieu group $M_{24}$ and the alternating group $A_{12}$ in characteristic 2. This new technique allows for further analysis of large finite simple groups and their representations. These results have direct applications in other areas of Algebra; for example, a known scheme for computing the Ext-algebra for simple groups and computing the cohomology groups of $FG$-modules via projective resolutions depends upon knowing the P.I.M.s of $G$. This method for calculating the P.I.M.s presents an opportunity to extend these techniques to larger groups. (Received September 23, 2009)