A variational principle is introduced to provide a new formulation and resolution for several boundary value problems. Indeed, we consider systems of the form

\[
\begin{align*}
\Lambda u &= \nabla \Phi(u), \\
\beta_2 u &= \nabla \Psi(\beta_1 u)
\end{align*}
\]

where \(\Phi\) and \(\Psi\) are two convex functions and \(\Lambda\) is a possibly unbounded self-adjoint operator modulo the boundary operator \(B = (\beta_1, \beta_2)\). We shall show that solutions of the above system coincide with critical points of the functional

\[
I(u) = \Phi^*(\Lambda u) - \Phi(u) + \Psi^*(\beta_2 u) - \Psi(\beta_1 u)
\]

where \(\Phi^*\) and \(\Psi^*\) are Fenchel-Legendre dual of \(\Phi\) and \(\Psi\) respectively. Note that the standard Euler-Lagrange functional corresponding to the system above is of the form,

\[
F(u) = \frac{1}{2} \langle \Lambda u, u \rangle - \Phi(u) - \Psi(\beta_1 u).
\]

An immediate advantage of using the functional \(I\) instead of \(F\), is to obtain more regular solutions and also the flexibility to handle boundary value problems with nonlinear boundary conditions. Applications to Hamiltonian systems and semi-linear Elliptic equations with various linear and nonlinear boundary conditions are also provided. (Received September 07, 2009)