Preperiodic points: from algebraic to complex.

We combine analytic and arithmetic techniques to study the dynamics of complex polynomials. For quadratic polynomials $z^2 + c$, our main result is that the set of parameters $c$ for which two given complex numbers $a$ and $b$ are both preperiodic is finite iff $a^2 \neq b^2$. This is a dynamical analog of recent results of Masser and Zannier concerning simultaneous torsion sections on families of elliptic curves. This is joint work with Matt Baker. (Received September 21, 2009)